UDC 551.524.36:551.501.45:519.24(788)

Spectral Analysis of Daily Maximum and Minimum Temperature Series on the East Slope of the Colorado Front Range

ERAT S. JOSEPH—*Lafayette College, Easton, Pa.*

ABSTRACT—Power spectral analysis of daily maximum and daily minimum temperature series at four stations on the east slope of the Colorado Front Range reveals the presence of dominant annual oscillations. Cross correlation and cross spectrum indicate that the series at these spatially separated stations are well correlated. The coherences between annual components are especially high and their phase difference is zero.

1. INTRODUCTION

A time series consisting of data collected in successive intervals of time is said to be randomly distributed if each observation is statistically independent of all preceding and succeeding observations. Many climatological series are characterized not only by significant serial correlation within the series, but also by pronounced mutual dependence between them. Though many of the statistical techniques based on the assumption of independence consider dependence as a nuisance, analysis of the nature of autocorrelation and cross correlation of series often yields extremely fruitful results.

Many past investigators such as Landsberg et al. (1959), Yevdjevich (1963), Matalas (1966), Polowchak and Panofsky (1968), Rodriguez-Iturbe and Nordin (1969), and others have demonstrated how spectral analysis helps to increase comprehension of the underlying characteristics of natural phenomena.

Time series can be analyzed either in the time domain or in the frequency domain. In the latter case, the series is considered to consist of a large number of periodic components. The relative importance of a periodic component is reflected by the power spectrum, which measures the amount of variance contributed by that component. When two series are involved, the interrelationship between pairs of frequency components is analyzed by a cross spectrum.

2. SPECTRAL THEORY

For a stationary series $X(t)$ with mean μ, the autocovariance function $\gamma_{XX}(u)$ and the autocorrelation function $\rho_{XX}(u)$ are functions of lag u; that is,

$$\gamma_{XX}(u)=E\{[X(t)-\mu][X(t+u)-\mu]\} \tag{1}$$

and

$$\rho_{XX}(u)=\frac{\gamma_{XX}(u)}{\gamma_{XX}(0)}. \tag{2}$$

The power spectrum $\Gamma_{XX}(\omega)$ of process $X(t)$ is the Fourier transform of the autocovariance function; that is,

$$\Gamma_{XX}(\omega)=\frac{1}{\pi}\int_{-\infty}^{\infty}\gamma_{XX}(u)e^{-i\omega u}du. \tag{3}$$

The power spectrum curve shows how the variance of the process is distributed with frequency. The variance of a process contributed by frequencies in the range of ω to $\omega+d\omega$ is given by the area under the power spectrum curve between ω and $\omega+d\omega$. A peak in the power spectrum is an indication of an important frequency band. Thus, a visual examination of the spectrum often reveals the important frequency components of the process.

The cross covariance function $\gamma_{XY}(u)$ and cross correlation function $\rho_{XY}(u)$ are measures of dependence at lag u between two processes $X(t)$ and $Y(t)$; that is,

$$\gamma_{XY}(u)=E\{[X(t)-\mu_X][Y(t+u)-\mu_Y]\} \tag{4}$$

and

$$\rho_{XY}(u)=\frac{\gamma_{XY}(u)}{\sqrt{\gamma_{XX}(0)\gamma_{YY}(0)}}. \tag{5}$$

Cross spectrum $\Gamma_{XY}(\omega)$ is the Fourier transform of the cross covariance function; that is,

$$\Gamma_{XY}(\omega)=\frac{1}{\pi}\int_{-\infty}^{\infty}\gamma_{XY}(u)e^{-i\omega u}du. \tag{6}$$

The cospectrum $c(\omega)$ and quadrature spectrum $q(\omega)$ are obtained by rewriting the cross spectrum as

$$\Gamma_{XY}(\omega)=c_{XY}(\omega)+iq_{XY}(\omega). \tag{7}$$

The concept that time series can be decomposed into many periodic components is useful while studying the relationship in the frequency domain between two processes. The correlation between components with equal frequencies is measured by the coherence $C_{XY}(\omega)$ at the respective frequencies; that is,

$$C_{XY}(\omega)=\frac{c^2_{XY}(\omega)+q^2_{XY}(\omega)}{\Gamma_{XX}(\omega)\Gamma_{YY}(\omega)}. \tag{8}$$

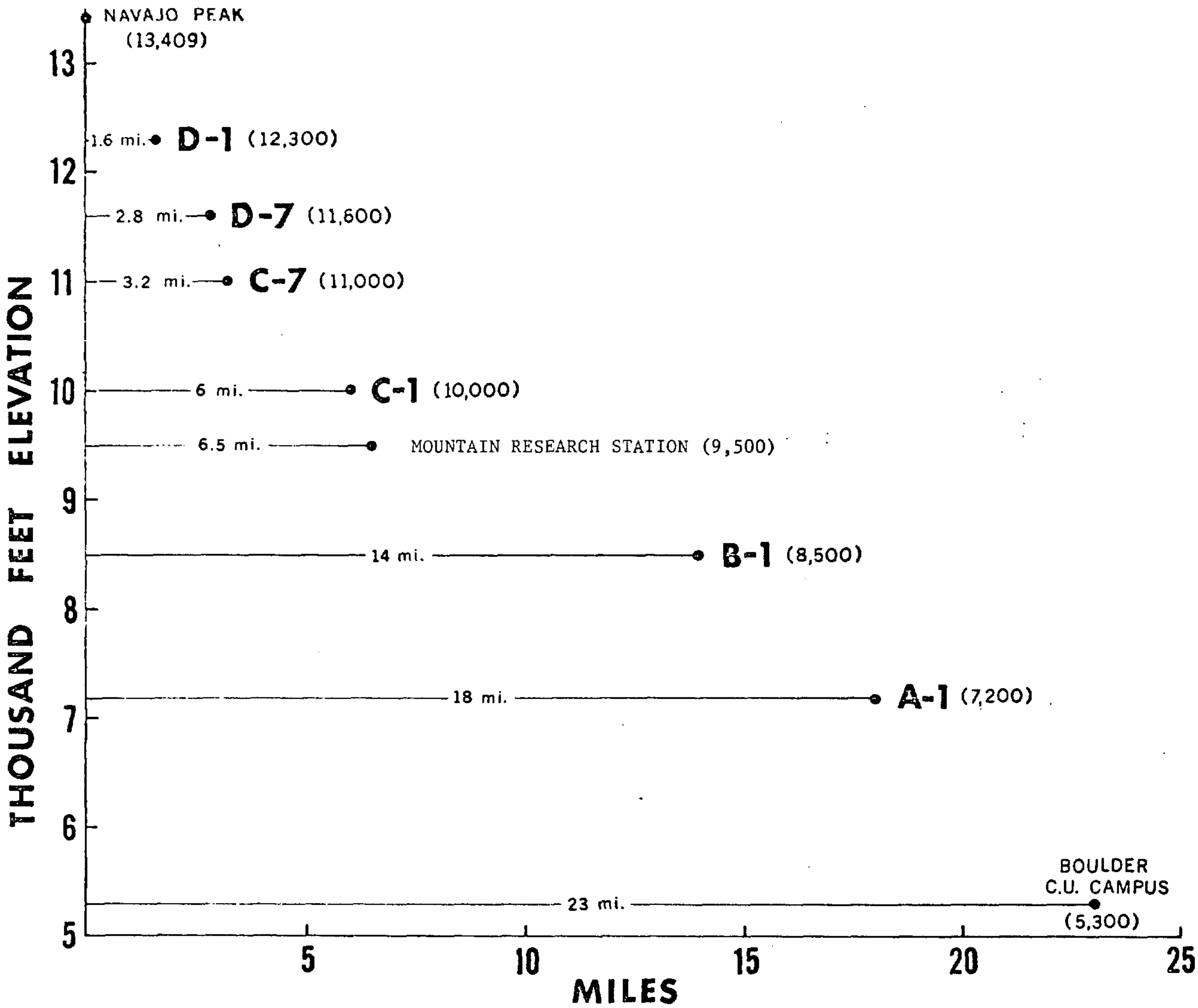

FIGURE 1.—Station elevations and distances from the Continental Divide.

The coherence spectrum plays a role equivalent to a correlation coefficient at each frequency. Both coherence and cross correlation functions measure the extent of correlation between two time series, the former at each frequency of the series and the latter at different lags of time.

The coherence spectrum, however, gives no indication of any phase difference between components of equal frequency of two series. Hence, to describe the frequency characteristics of a bivariate process, one needs a phase spectrum as well as a coherence spectrum. Thus, phase and coherence diagrams are powerful tools for visually examining the relationship between two processes component by component. Phase difference $\phi_{XY}(\omega)$ between two processes $X(t)$ and $Y(t)$ at frequency ω is given by

$$\phi_{XY}(\omega)=\arctan \frac{q_{XY}(\omega)}{c_{XY}(\omega)}. \tag{9}$$

A straightline phase diagram through the origin indicates that the two series differ by a fixed simple time lag equal to the slope of the phase diagram. Quite often, the phase function may represent more complex lead or lag relationships. The phase diagram has to be studied in conjunction with the coherence diagram; those parts of the phase diagram where coherence is high are more important than those frequencies with nonsignificant coherence.

3. ANALYSIS OF DATA

The eight series analyzed are T_{max}A, T_{max}B, T_{max}C, T_{max}D, T_{min}A, T_{min}B, T_{min}C, and T_{min}D. They consist of daily maximum and daily minimum temperatures at stations A,B,C,D (elevations 7,200, 8,500, 10,000, and 12,300 ft, respectively) on the east slope of the Colorado Front Range. Figure 1 shows the elevation of the stations above mean sea level and their distance from the continental divide.

In this investigation, each time series consisted of 2,099 observations. The correlation functions and spectra were estimated up to a maximum lag of 1 yr. Although there is no one accepted rule regarding the value of the maximum lag, m, many statisticians consider it adequate to estimate the spectrum up to a maximum lag equal to one-tenth of the length of series. Granger and Hatanaka (1964) state that, with n pieces of data, only rarely should m be as

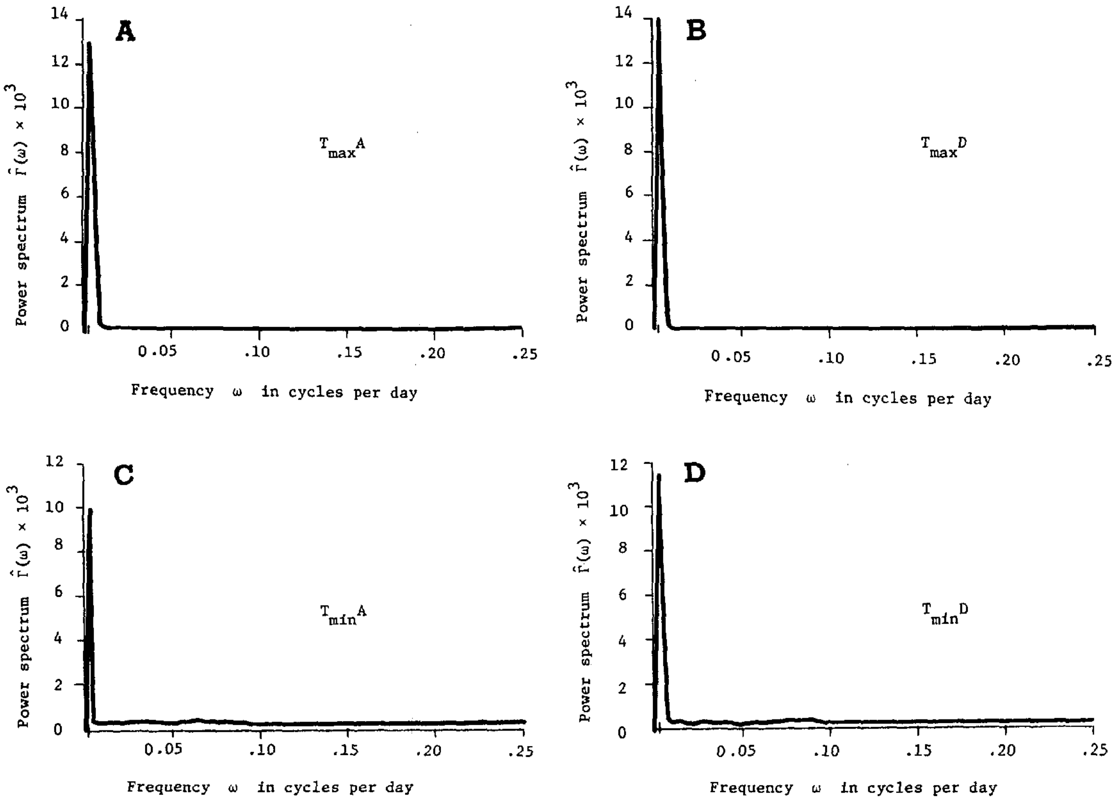

FIGURE 2.—Power spectra of T_{max} at (A) station A and (B) station D and of T_{min} at (C) station A and (D) station D.

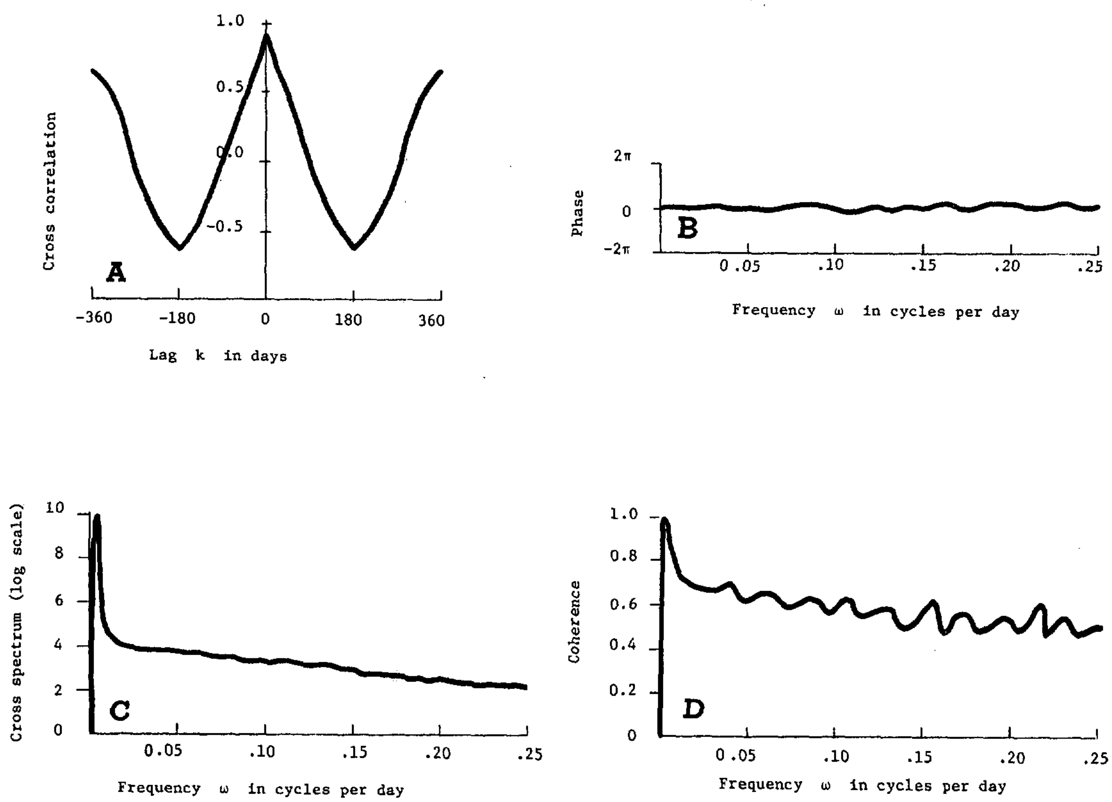

FIGURE 3.—The (A) cross correlation, (B) phase differences, (C) cross spectra, and (D) coherence between T_{max} at stations D and A.

large as $n/3$; for n not large, m on the order of $n/5$ or $n/6$ would seem to be reasonable. Jenkins and Watts (1969) propose an empirical approach based on computing the spectra for various m values and selecting the one for which the variance and the bias are reasonably small. While this approach is certainly desirable, it requires more computing time. Since the annual cycle was of special importance, a maximum lag of 1 yr was used in this study.

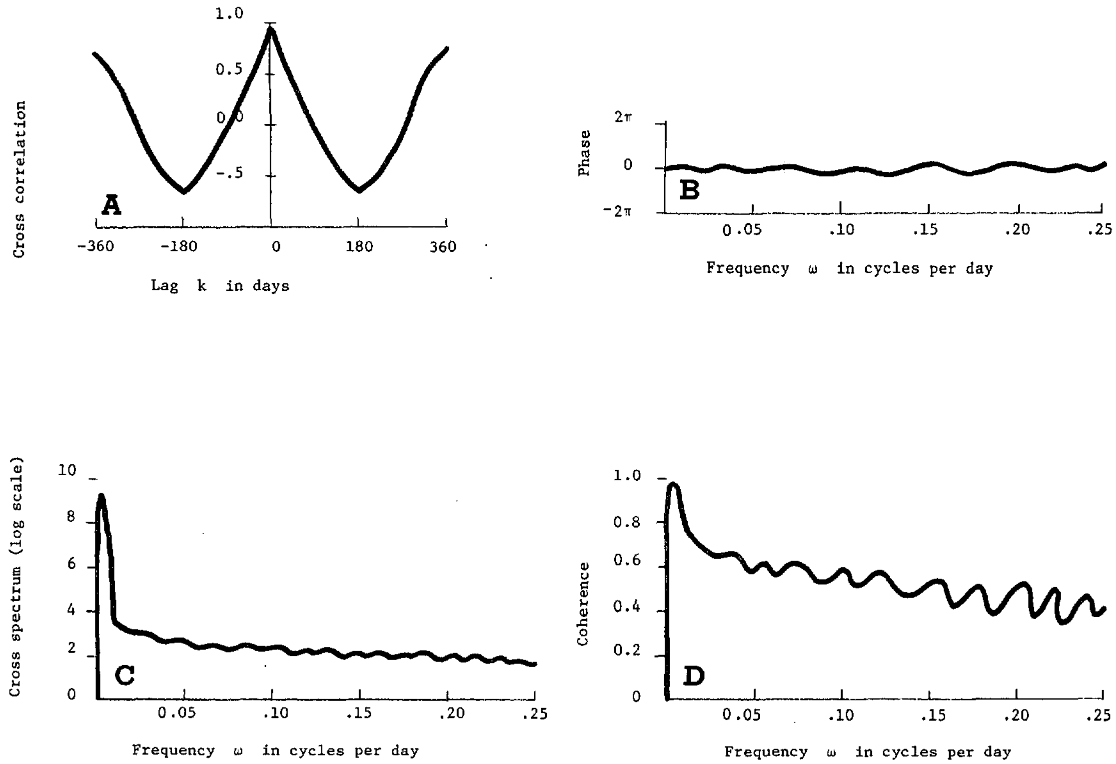

FIGURE 4.—Same as figure 3 for T_{min} at stations D and A.

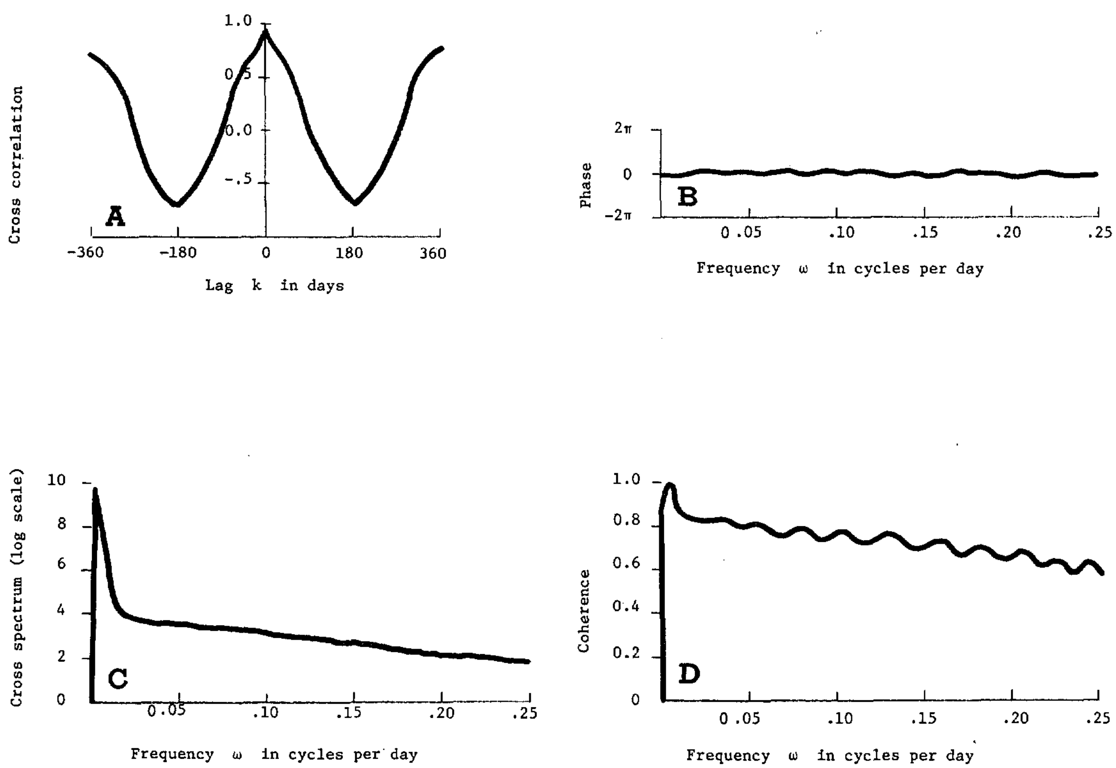

FIGURE 5.—Same as figure 3 for T_{min} and T_{max} at station D.

Figures 2–6 graphically present some of the power spectra of individual series and the cross correlation function, cross spectrum, phase diagram, and coherence diagram of pairs of series. Visual examination of these diagrams reveals many characteristics of daily temperature extremes on the east slope of the Colorado Front Range.

4. CONCLUSIONS

The general conclusions are the following:

1. Power spectra of all series have sharp peaks at the annual frequency. There are no other well-defined peaks at higher frequencies. Thus, while the annual cycle is an important component,

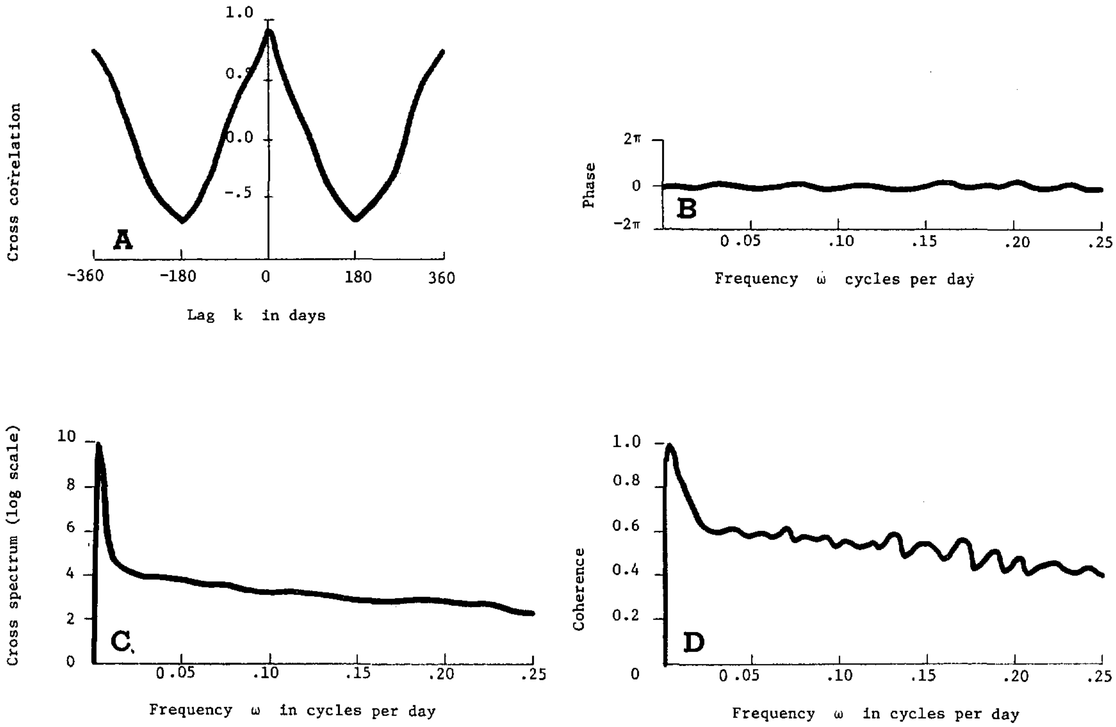

FIGURE 6.—Same as figure 3 for T_{max} at station A and T_{min} at station D.

there is no indication of any important shorter cycles. It should be emphasized that the time series analyzed is not long enough to draw any conclusion whatever regarding low-frequency components with periods exceeding 1 yr.

2. Table 1 shows the coherences and phase differences between series belonging to stations A and D, which are farthest from each other among the four stations. The phase differences between these series are nearly zero for low frequencies including the annual component. The coherences at low frequencies are high and are highest at the annual frequency. Thus, while the power spectra reveal the existence of dominant annual cycles, the cross spectra indicate the strong correlations among the annual components of the various series at stations spatially separated. Also, the phase diagrams indicate that the annual components have no significant lead or lag.

3. The cross correlation functions of all pairs of series have their maximums at zero lag. These maximums lie between 0.91 and 0.97, indicating that concurrent daily extremes of temperatures at the different stations are well correlated.

4. The significant autocorrelation within all series and the cross correlation among the series are indications of the grouping tendency of daily temperature extremes for many consecutive days over the east slope of the Front Range. The fact that the series are spatially correlated is ample evidence to reject the hypothesis that the daily temperature extremes at each station in the Front Range depend mainly on local factors independent of other stations. The close relationship between series makes it possible to construct one series from another and thus enables the estimation of missing observations in one series from the known observations of another series.

ACKNOWLEDGMENTS

The author is indebted to Roger G. Barry, University of Colorado, for giving his valuable suggestions. The data bank at INSTAAR (which is being developed under NSF Grant GA 15528) and other facilities at the Institute of Arctic and Alpine Research were provided by Jack D. Ives. Financial support for this research was provided by the National Science Foundation under the Research Participation program for college teachers—1972.

TABLE 1.—*Cross correlation and cross spectral characteristics of annual components of indicated pairs of series*

Series	Cross correlation coefficient at zero lag	Coherence at annual frequency	Phase in fraction of a circle at annual frequency
T_{max}D vs. T_{max}A	0. 92	0. 95	0. 001
T_{min}D vs. T_{min}A	. 94	. 98	. 003
T_{min}D vs. T_{max}D	. 96	. 90	. 007
T_{min}D vs. T_{max}A	. 91	. 87	. 008

REFERENCES

Granger, C. W. J., and Hatanaka, M., *Spectral Analysis of Economic Time Series*, Princeton University Press, New Jersey, **1964**, 299 pp. (See pp. 1–262.)

Jenkins, Gwilym M., and Watts, Donald G., *Spectral Analysis and its Applications*, Holden-Day Co., New York, N.Y., **1969**, 525 pp. (See pp. 1–457.)

Landsberg, H. E., Mitchell, J. M., Jr., and Crutcher, H. L., "Power Spectrum Analysis of Climatological Data for Woodstock College, Maryland," *Monthly Weather Review*, Vol. 87, No. 8, Aug. **1959**, pp. 283–298.

Matalas, N. C., "Some Aspects of Time Series Analysis in Hydrologic Studies," *Proceedings of the Hydrology Symposium No. 5 McGill University, Montreal, Canada*, McGill University, **1966**, pp. 271–309.

Polowchak, Van M., and Panofsky, Hans A., "The Spectrum of Daily Temperatures as a Climatic Indicator," *Monthly Weather Review*, Vol. 96, No. 9, Sept. **1968**, pp. 596–600.

Rodriguez-Iturbe, Ignacio, and Nordin, Carl F., "Some Applications of Cross-Spectral Analysis in Hydrology: Rainfall and Runoff," *Water Resources Research*, Vol. 5, No. 3, June **1969**, pp. 608–621.

Yevdjevich, V. M., "Fluctuations of Wet and Dry Years," *Hydrology Paper* no. 1, Colorado State University, Fort Collins, July **1963**, pp. 1–55.

[*Received March 1, 1973*]